Ranking and importance in complex networks

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Introduction

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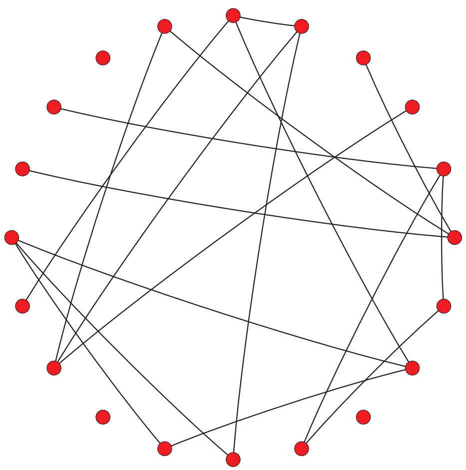
Rough structure of this talk:

- Graph theory *without tears*
  - Basic notions
  - Random graphs
  - Ranking schemes
- Robustness of the ranking schemes
  - Perturbations
  - Deviation measures
  - Results
- Application to real data
  - Datasets
  - Identifying essentiality
  - Results



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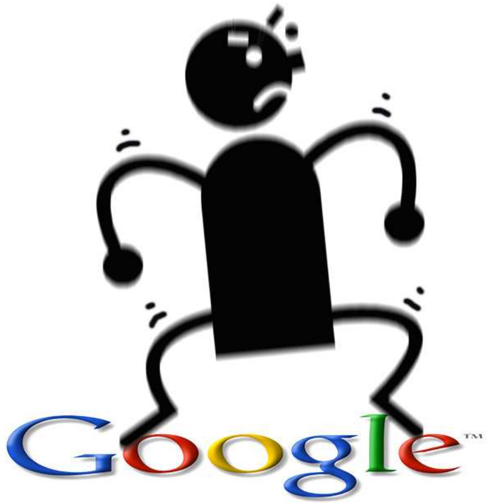

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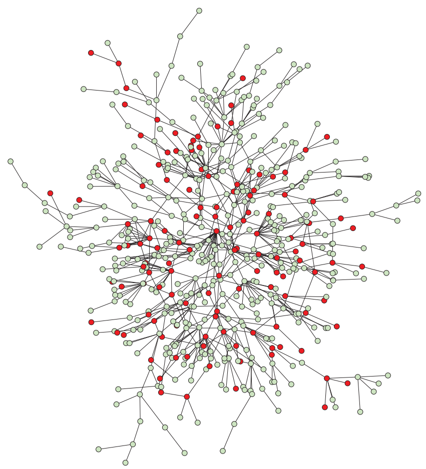

Graph theory

Introduction

- Idea by Euler, ~ 1730

  $\vdots$

- Some of the applications today: modeling of
  - Internet, WWW, Intranets
  - Social networks, epidemics
  - Infrastructures (roads, power grids)
  - Phone call networks
  - Collaboration networks
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Basic notions

- What is a graph?
  - Loosely speaking: *A graph is a bunch of nodes connected by a bunch of edges*
  - More formally: $\mathscr{G} = (\mathcal{V}, \mathcal{E})$ ...
  - Description via matrices: adjacency–, distance–, Laplacian– & incidence–matrix
- Characteristics
  - Undirected / directed
  - Connected / disconnected
  - Characteristic values: number of nodes / edges, average node degree, degree distribution, average path length, diameter, clustering coefficient, ...

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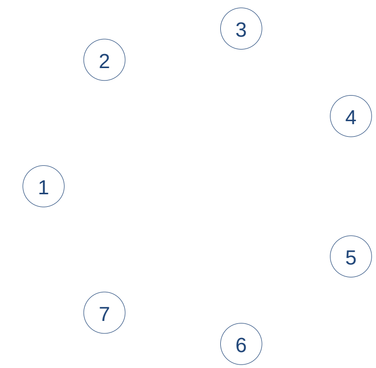

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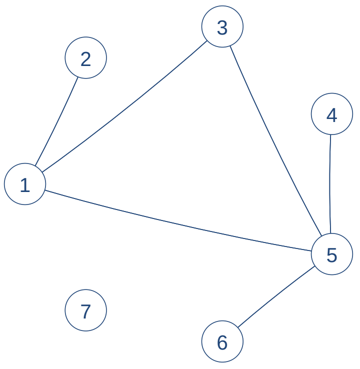

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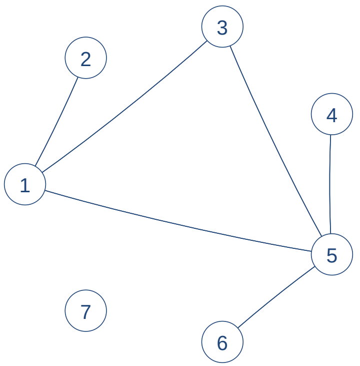

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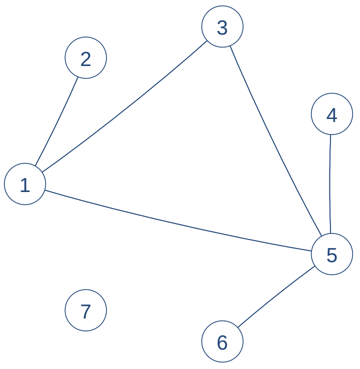

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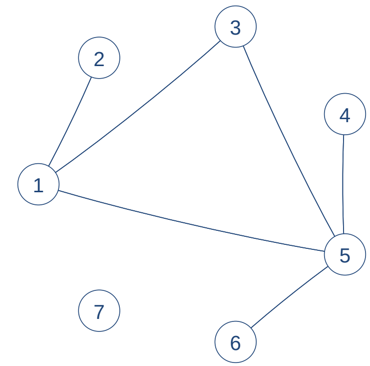

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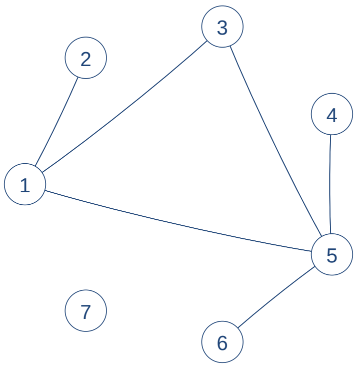

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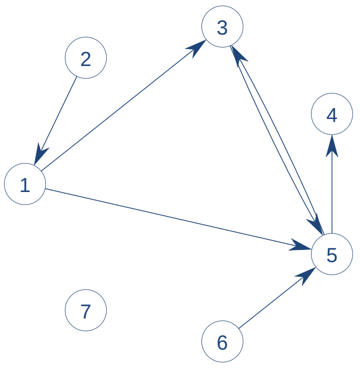

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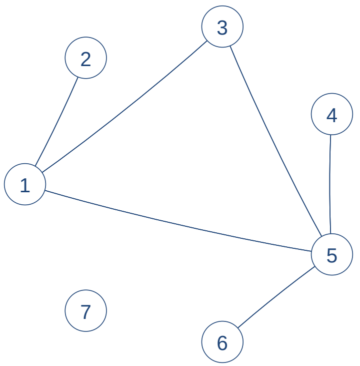

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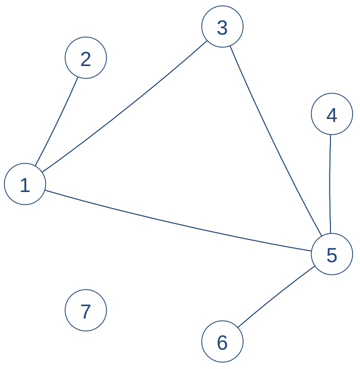

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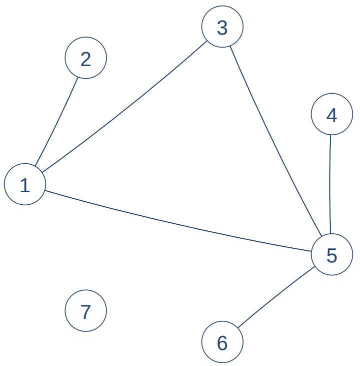

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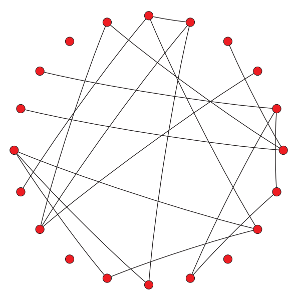
Random graphs

We looked at 3 models:

- Erdös–Rényi (classical), $\sim$ 1957
  - Seminal, first model
  - No “order”
  - Few practical applications



- Watts–Strogatz (small–world) $\sim$ 1999
  - Model for social networks
  - Highly clustered, rel. short paths
  - Used in sociology, biology, . . .
- Barabási–Albert (scale–free) $\sim$ 1999
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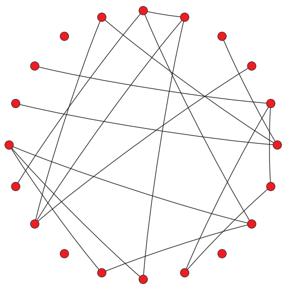

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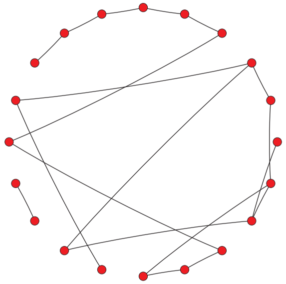

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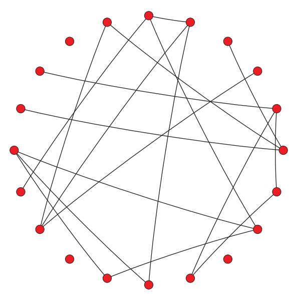

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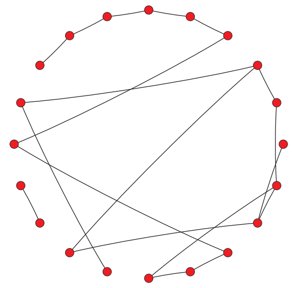

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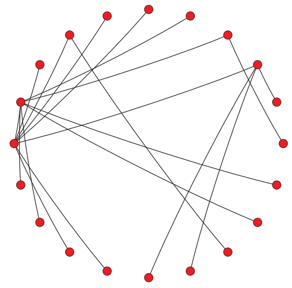

Ranking schemes

- Growing complexity of networks — *Who's important?*

  $\Rightarrow$ Ranking of the nodes
- Most immediate
  - Node degrees (ND)
- Eigenvector based
  - HITS
  - PageRank (PR)
- Centrality based
  - Excentricity (EXC)
  - Status (STA)
  - Centroid value (CV)
- Impact on topology based
  - Damage (DAM)

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Place	Score	Node
1.	0.2568	G
2.	0.1644	A
3.	0.1118	C
4.	0.1053	B
5.	0.0625	E
6.	0.0616	F
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Robustness

Introduction

- What impact have perturbations on rankings?
- Real data in many cases noisy / missing:
  - Inaccurate measuring
  - Deliberate sampling
- 6 deviation measures:
  - 3 based on node ranks
  - 3 based on top percentile of nodes
- Types of perturbation:
  - Edge removal / rewiring / addition
  - Node removal
- Simulations:
  1. Generate & rank BA graph
  2. Perturb graph
  3. Rank perturbed graph
  4. Compare rankings

Place	Node
1.	C
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Deviation measures

- Based on node ranks:

Original		Perturbed	
Rank	Node	Rank	Node
1.	C		
2.	H		
3.	D		
4.	J		
5.	A		
.	.		
.	.		
.	.		
.	.		
.	.		

- Based on top fraction of nodes:

Deviation measures

- Based on node ranks:

Original		Perturbed	
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3.	D	.	.
4.	J	4	J
5.	A	.	.
.	.	6	C
.	.	.	.
.	.	.	.
.	.	9	D
.	.	.	.

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$$\delta_1 = \underbrace{|1-6|}_{\text{Node C}} + \underbrace{|2-2|}_{\text{Node H}} + \underbrace{|3-9|}_{\text{Node D}} + \underbrace{|4-4|}_{\text{Node J}} + \underbrace{|5-1|}_{\text{Node A}} = 15$$

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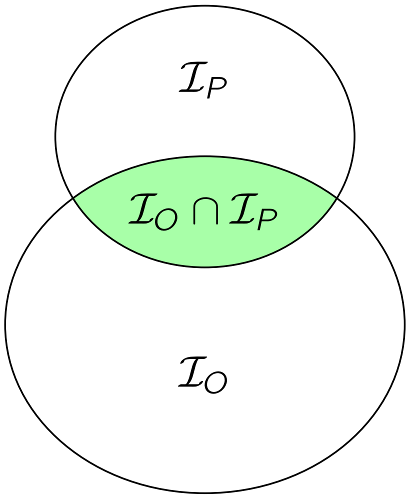
·	·	9	D
·	·	·	·

$$\delta_1 = \underbrace{|1-6|}_{\text{Node C}} + \underbrace{|2-2|}_{\text{Node H}} + \underbrace{|3-9|}_{\text{Node D}} + \underbrace{|4-4|}_{\text{Node J}} + \underbrace{|5-1|}_{\text{Node A}} = 15$$

- Based on top fraction of nodes:



Deviation measures

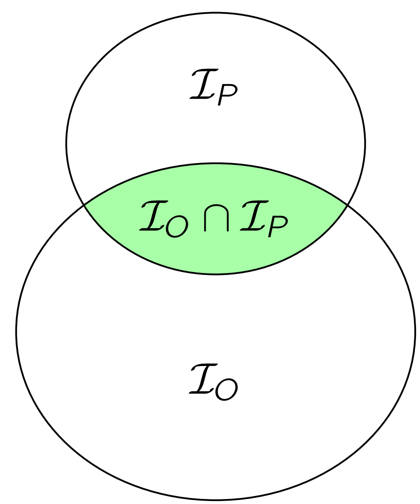
- Based on node ranks:

$$\delta_1 = \underbrace{|1-6|}_{\text{Node C}} + \underbrace{|2-2|}_{\text{Node H}} + \underbrace{|3-9|}_{\text{Node D}} + \underbrace{|4-4|}_{\text{Node J}} + \underbrace{|5-1|}_{\text{Node A}} = 15$$

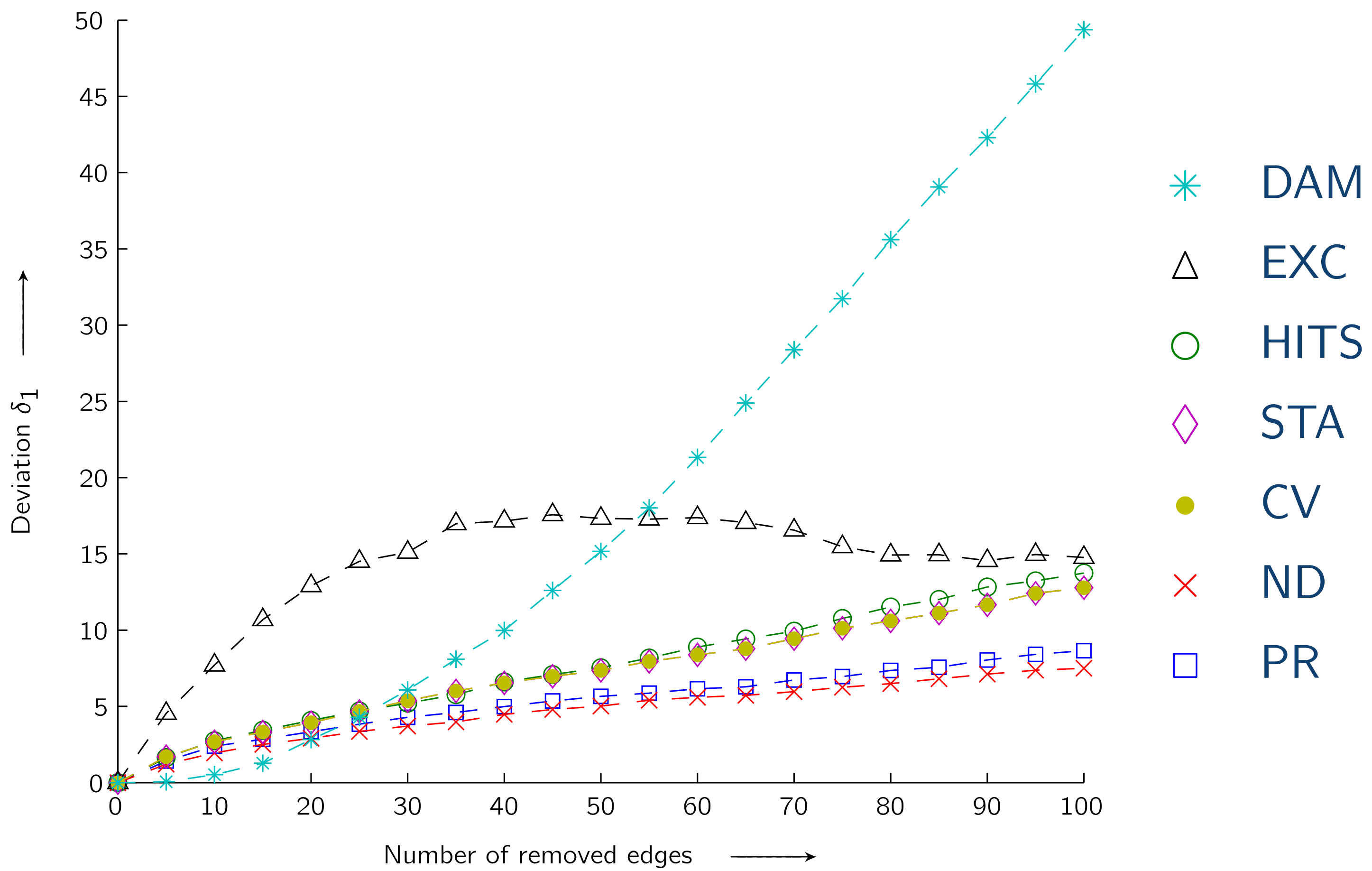
Original		**Perturbed**	
Rank	Node	Rank	Node
1.	C	1	A
2.	H	2	H
3.	D	·	·
4.	J	4	J
5.	A	·	·
·	·	6	C
·	·	·	·
·	·	·	·
·	·	9	D
·	·	·	·

- Based on top fraction of nodes:

$$\delta_4 = 1 - \frac{|\mathcal{I}_O \cap \mathcal{I}_P|}{|\mathcal{I}_P|}$$

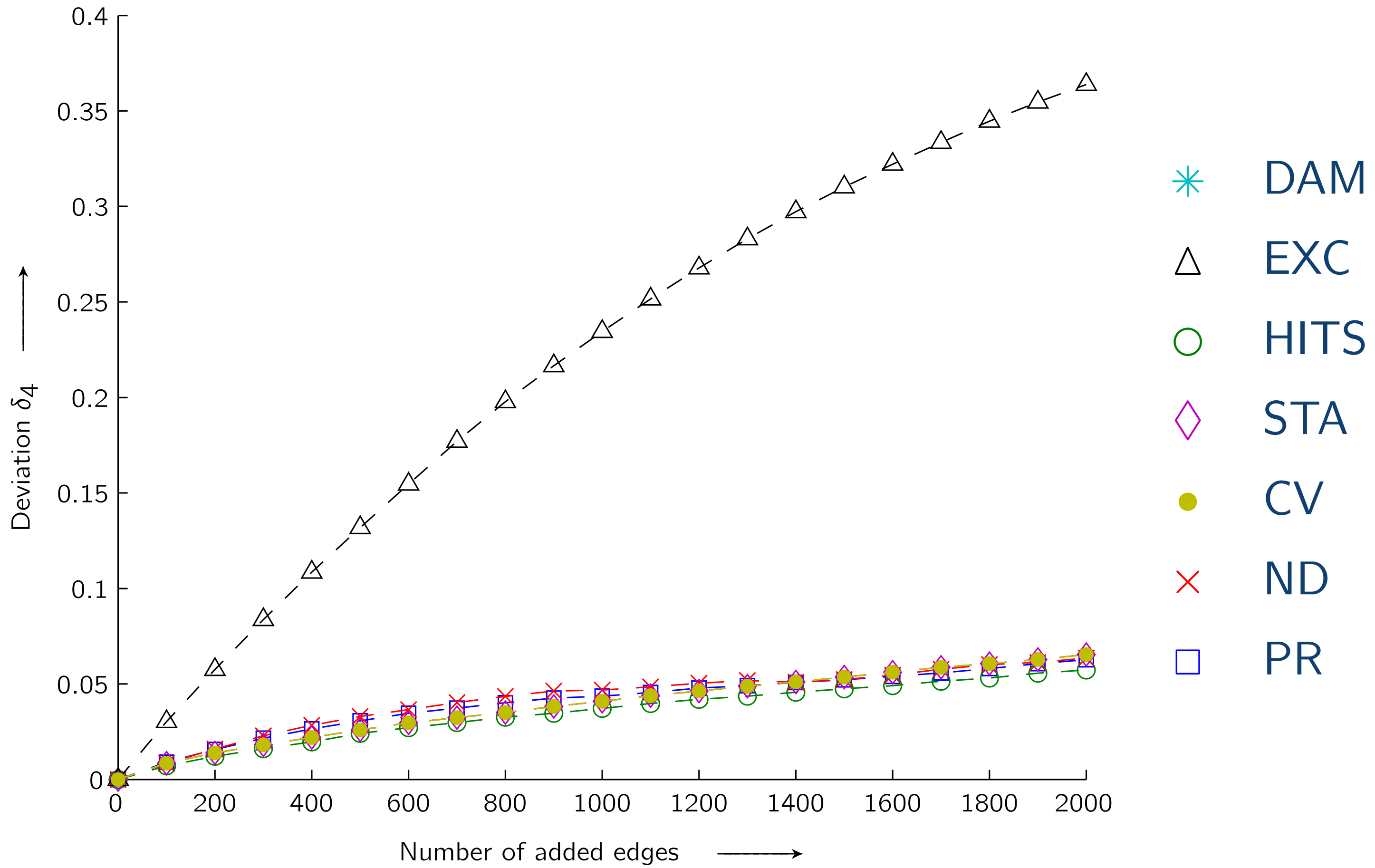


Results



Average value of 250 Barabási–Albert graphs on 150 nodes and 450 edges

Results



Average value of 250 Barabási–Albert graphs on 3000 nodes and 9000 edges

Results

- PR and ND often most robust
- However, few cases where HITS, CV or STA better
- EXC sensitive + useless when graph disconnected
- DAM relatively robust, but costly
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  - Interpretation of “deviation”
  - Type of perturbations
  - Resolution of the ranking needed
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Real data

Introduction

- Nodes' ranked importance $\overset{?}{\leftrightsquigarrow}$ real importance ?

- Protein–Protein interaction networks
  - Undirected graphs
  - Isolate largest connected component
  - Top–ranked–nodes $\overset{?}{\leftrightsquigarrow}$ Essential–for–survival

- Measure of success:

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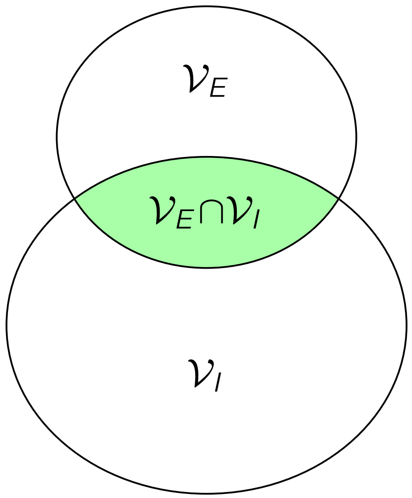

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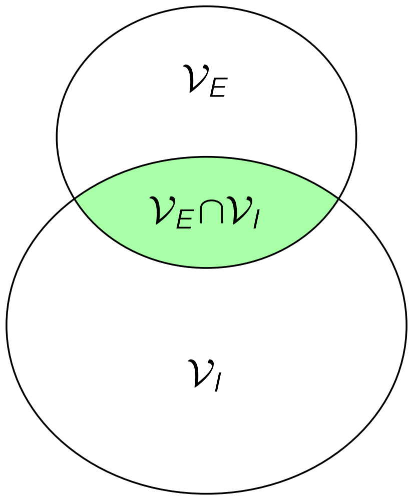

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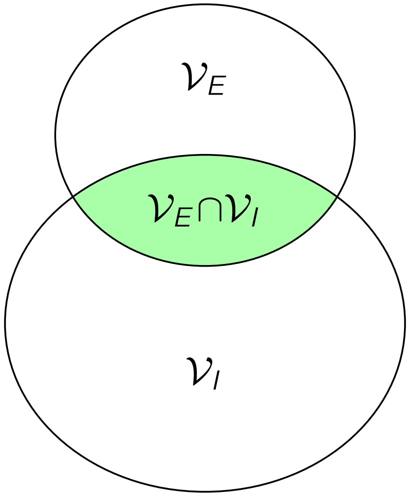

Datasets

Protein–protein interaction network of the budding yeast *Saccharomyces cerevisiae*.

- Uetz *et al.* [1)]
  - $n = 558, \quad e = 646$
  - $h_{ess} = 22.6\%$
- Ito *et al.* [2)]
  - $n = 2840, \quad e = 4147$
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2) *Proc. of the Nat. Academy of Sciences of the USA*, 98:4569–4574, April 2001
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Introduction

Graph theory

Robustness

Real data
· Introduction
· Datasets
· Identif. essent.
· Robustness

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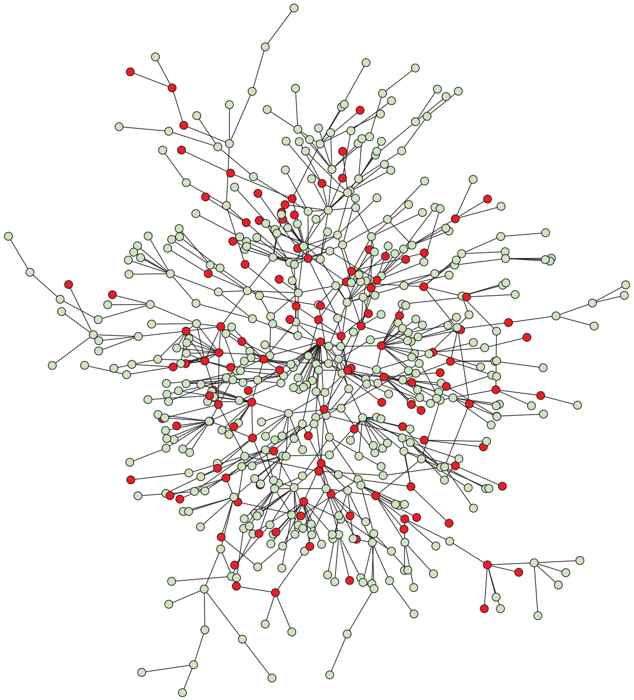

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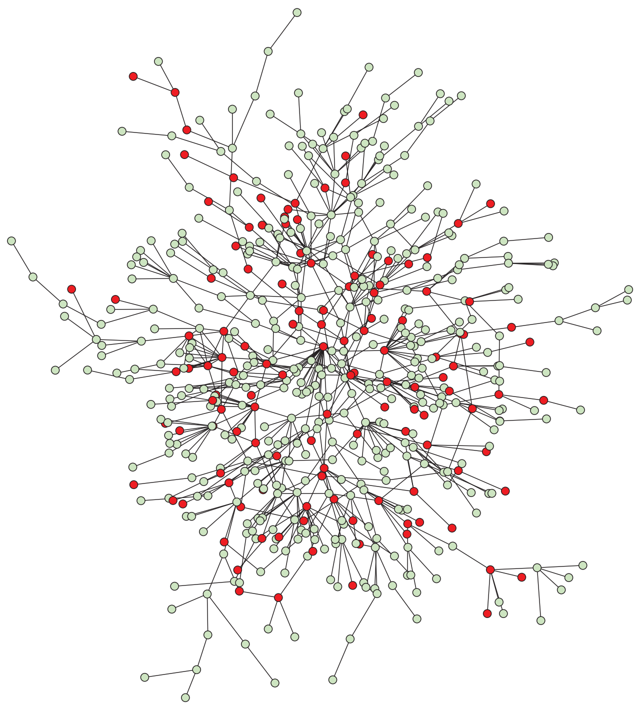

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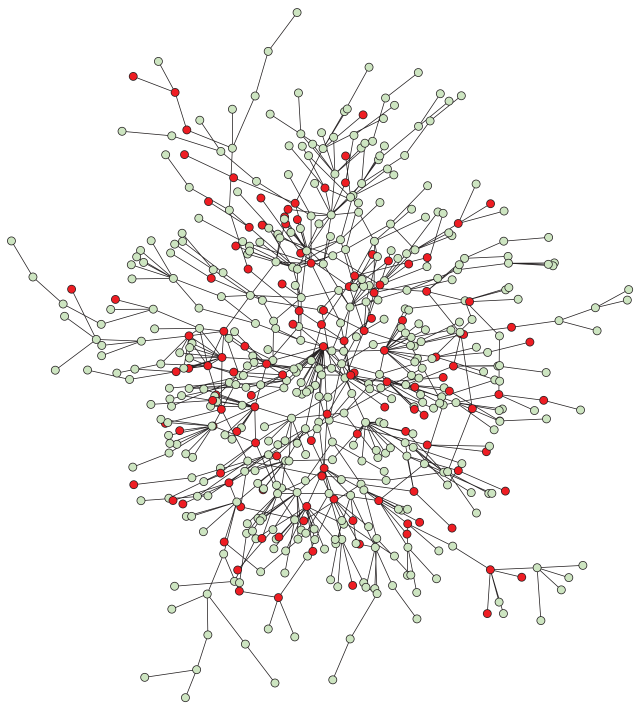

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Identifying essentiality

- We considered the top ranked 5% of nodes as "important"
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- STA & CV mixed results
- Further investigations needed for DAM

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HITS	35.7	16.6	43.8
PR	46.4	**24.8**	**62.1**
EXC	24.6	18.4	34.7
STA	17.2	19.7	59.5
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DAM	44.0	?	?
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- My family and close friends

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